

Almost contra  $v$ -closed mappingsBalasubramanian S<sup>1</sup>, Aruna Swathi Vyjayanthi P<sup>2\*</sup>, Sandhya C<sup>3</sup>

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## ABSTRACT

The aim of this paper is to introduce and study the concept of Almost contra  $v$ -closed mappings and the interrelationship between other Almost contra-closed maps. **Keywords:**  $v$ -open set,  $v$ -open map,  $v$ -closed map, Almost contra-closed map, Almost contra-pre closed map and Almost contra  $v$ -closed map.

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## 1. INTRODUCTION

Mappings plays an important role in the study of modern mathematics, especially in Topology and Functional analysis. Closed mappings are one such mappings which are studied for different types of closed sets by various mathematicians for the past many years. N.Biswas, discussed about semiopen mappings in the year 1970, A.S.Mashhour, M.E.Abd El-Monsef and S.N.El-Deeb studied preopen mappings in the year 1982 and S.N.El-Deeb, and I.A.Hasanien defined and studied about preclosed mappings in the year 1983. Further Asit kumar sen and P. Bhattacharya discussed about pre-closed mappings in the year 1993. A.S.Mashhour, I.A.Hasanein and S.N.El-Deeb introduced  $\alpha$ -open and  $\alpha$ -closed mappings in the year in 1983, F.Cammaroto and T.Noiri discussed about semipre-open and semipre-closed mappings in the year 1989 and G.B.Navalagi further verified few results about semipreclosed mappings. M.E.Abd El-Monsef, S.N.El-Deeb and R.A.Mahmoud introduced  $\beta$ -open mappings in the year 1983 and Saeid Jafari and T.Noiri, studied about  $\beta$ -closed mappings in the year 2000. C. W. Baker, introduced Contra-open functions and contra-closed functions in the year 1997. M.Caldas and C.W.Baker introduced contra pre-semiopen Maps in the year 2000. In the year 2010, S. Balasubramanian and P.A.S.Vyjayanthi introduced  $v$ -open mappings and in the year 2011 they further defined almost  $v$ -open mappings. In the last year S. Balasubramanian and P.A.S.Vyjayanthi introduced  $v$ -closed and Almost  $v$ -closed mappings. Inspired with these concepts and its interesting properties we in this paper tried to study a new variety of closed maps called contra  $v$ -closed maps. Throughout the paper  $X$ ,  $Y$  means topological spaces  $(X, \tau)$  and  $(Y, \sigma)$  on which no separation axioms are assured.

## 2. PRELIMINARIES

**Definition 2.1:**  $A \subseteq X$  is said to be

- regular open[pre-open; semi-open;  $\alpha$ -open;  $\beta$ -open] if  $A = \text{int}(\text{cl}(A))$  [ $A \subseteq \text{int}(\text{cl}(A))$ ;  $A \subseteq \text{cl}(\text{int}(A))$ ;  $A \subseteq \text{int}(\text{cl}(\text{int}(A)))$ ;  $A \subseteq \text{cl}(\text{int}(\text{cl}(A)))$ ] and regular closed[pre-closed; semi-closed;  $\alpha$ -closed;  $\beta$ -closed] if  $A = \text{cl}(\text{int}(A))$  [ $\text{cl}(\text{int}(A)) \subseteq A$ ;  $\text{int}(\text{cl}(A)) \subseteq A$ ;  $\text{cl}(\text{int}(\text{cl}(A))) \subseteq A$ ;  $\text{int}(\text{cl}(\text{int}(A))) \subseteq A$ ]
- $v$ -open if there exists regular-open set  $U$  such that  $U \subseteq A \subseteq \text{cl}(U)$ .
- $g$ -closed[ $rg$ -closed] if  $\text{cl}(A) \subseteq U$  [  $\text{rcl}(A) \subseteq U$ ] whenever  $A \subseteq U$  and  $U$  is open[ $r$ -open] in  $X$  and  $g$ -open[ $rg$ -open] if its complement  $X - A$  is  $g$ -closed[ $rg$ -closed].

**Remark 1:** We have the following implication diagrams for closed sets.
$$\begin{array}{ccccccc}
 r\alpha\text{-closed set} & \rightarrow & r\alpha\text{-closed set} & \rightarrow & v\text{-closed set} & & \\
 \downarrow & & & & \downarrow & & \\
 \text{pre-closed set} & \leftarrow & \text{closed set} & \rightarrow & \alpha\text{-closed set} & \rightarrow & \text{semi-closed set} \rightarrow \beta\text{-closed set.}
 \end{array}$$
**Definition 2.2:** A function  $f: X \rightarrow Y$  is said to be

- continuous[resp: semi-continuous,  $r$ -continuous,  $v$ -continuous] if the inverse image of every open set is open [resp: semi open, regular open,  $v$ -open].
- irresolute [resp:  $r$ -irresolute,  $v$ -irresolute] if the inverse image of every semi open [resp: regular open,  $v$ -open] set is semi open [resp: regular open,  $v$ -open].
- closed[resp: semi-closed,  $r$ -closed] if the image of every closed set is closed [resp: semi closed, regular closed].

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- d)  $g$ -continuous [resp:  $rg$ -continuous] if the inverse image of every closed set is  $g$ -closed. [resp:  $rg$ -closed].  
 e) contra closed [resp: contra semi-closed; contra pre-closed; contra  $r\alpha$ -closed] if the image of every closed set in  $X$  is open [resp: semi-open; pre-open;  $r\alpha$ -open] in  $Y$ .

**Definition 2.3:**  $X$  is said to be  $T_{1/2}[r-T_{1/2}]$  if every (regular) generalized closed set is (regular) closed.

### 3. ALMOST CONTRA $v$ -CLOSED MAPPINGS

**Definition 3.1:** A function  $f: X \rightarrow Y$  is said to be almost contra  $v$ -closed if the image of every closed set in  $X$  is  $v$ -open in  $Y$ .

**Theorem 3.1:** Every almost contra  $r\alpha$ -closed map is almost contra  $v$ -closed but not conversely.

**Proof:** Let  $A \subseteq X$  be closed  $\Rightarrow f(A)$  is  $r\alpha$ -open in  $Y$  since  $f: X \rightarrow Y$  is almost contra  $r\alpha$ -closed  $\Rightarrow f(A)$  is  $v$ -open in  $Y$  since every  $r\alpha$ -open set is  $v$ -open. Hence  $f$  is almost contra  $v$ -closed.

**Example 1:** Let  $X = Y = \{a, b, c\}$ ;  $\tau = \{\emptyset, \{a\}, \{b, c\}, X\}$ ;  $\sigma = \{\emptyset, \{a\}, \{b\}, \{a, b\}, Y\}$ . Let  $f: X \rightarrow Y$  be defined  $f(a) = b$ ,  $f(b) = c$  and  $f(c) = a$ . Then  $f$  is almost contra  $v$ -closed, almost contra semi-closed, almost contra  $r\alpha$ -closed and almost contra  $\beta$ -closed but not almost contra closed, almost contra pre-closed, almost contra  $r$ -closed, almost contra  $\alpha$ -closed and almost contra  $rp$ -closed.

**Example 2:** Let  $X = Y = \{a, b, c\}$ ;  $\tau = \{\emptyset, \{a\}, \{b\}, \{a, b\}, X\}$ ;  $\sigma = \{\emptyset, \{a\}, \{b\}, \{a, b\}, Y\}$ . Let  $f: X \rightarrow Y$  be defined  $f(a) = b$ ,  $f(b) = c$  and  $f(c) = a$ . Then  $f$  is not almost contra  $v$ -closed.

**Theorem 3.2:** Every almost contra  $r$ -closed map is almost contra  $v$ -closed but not conversely.

**Proof:** Let  $A \subseteq X$  be closed  $\Rightarrow f(A)$  is  $r$ -open in  $Y$  since  $f: X \rightarrow Y$  is almost contra  $r$ -closed  $\Rightarrow f(A)$  is  $v$ -open in  $Y$  since every  $r$ -open set is  $v$ -open. Hence  $f$  is almost contra  $v$ -closed.

**Theorem 3.3:** Every almost contra  $v$ -closed map is almost contra semi-closed but not conversely.

**Proof:** Let  $A \subseteq X$  be closed  $\Rightarrow f(A)$  is  $v$ -open in  $Y$  since  $f: X \rightarrow Y$  is almost contra  $v$ -closed  $\Rightarrow f(A)$  is semi-open in  $Y$  since every  $v$ -open set is semi-open. Hence  $f$  is almost contra semi-closed.

**Theorem 3.4:** Every almost contra  $v$ -closed map is almost contra  $\beta$ -closed but not conversely.

**Proof:** Let  $A \subseteq X$  be closed  $\Rightarrow f(A)$  is  $v$ -open in  $Y$  since  $f: X \rightarrow Y$  is almost contra  $v$ -closed  $\Rightarrow f(A)$  is  $\beta$ -open in  $Y$  since every  $v$ -open set is  $\beta$ -open. Hence  $f$  is almost contra  $\beta$ -closed.

**Note 1:**

- almost contra closed maps and almost contra  $v$ -closed maps are independent of each other.
- almost contra  $\alpha$ -closed map and almost contra  $v$ -closed map are independent of each other.
- almost contra pre closed map and almost contra  $v$ -closed map are independent of each other.

**Note 2:** We have the following implication diagram among the open maps.

almost contra  $r$ -closed  $\rightarrow$  almost contra  $r\alpha$ -closed  $\rightarrow$  almost contra  $v$ -closed  
 $\downarrow$   $\downarrow$   
 almost contra pre-closed  $\leftarrow$  almost contra closed  $\rightarrow$  almost contra  $\alpha$ -closed  $\rightarrow$  almost contra semi-closed  $\rightarrow$  almost contra  $\beta$ -closed. None is reversible.

**Theorem 3.5:** If  $R\alpha O(Y) = vO(Y)$  then  $f$  is almost contra  $r\alpha$ -closed iff  $f$  is almost contra  $v$ -closed.

**Proof:** Follows from theorem 3.1

Conversely Let  $A \subseteq X$  be closed  $\Rightarrow f(A)$  is  $v$ -open in  $Y$  since  $f: X \rightarrow Y$  is Almost contra  $v$ -closed  $\Rightarrow f(A)$  is  $r\alpha$ -open in  $Y$  since every  $v$ -open set is  $r\alpha$ -open. Hence  $f$  is Almost contra  $r\alpha$ -closed.

**Theorem 3.6:** If  $vO(Y) = RO(Y)$  then  $f$  is Almost contra  $r$ -closed iff  $f$  is Almost contra  $v$ -closed.

**Proof:** Follows from theorem 3.2

Conversely Let  $A \subseteq X$  be closed  $\Rightarrow f(A)$  is  $v$ -open in  $Y$  since  $f: X \rightarrow Y$  is Almost contra  $v$ -closed  $\Rightarrow f(A)$  is  $r$ -open in  $Y$  since every  $v$ -open set is  $r$ -open. Hence  $f$  is Almost contra  $r$ -closed.

**Theorem 3.7:** If  $vO(Y) = \alpha O(Y)$  then  $f$  is Almost contra  $\alpha$ -closed iff  $f$  is Almost contra  $v$ -closed.

**Proof:** Let  $A \subseteq X$  be closed  $\Rightarrow f(A)$  is  $\alpha$ -open in  $Y$  since  $f: X \rightarrow Y$  is Almost contra  $\alpha$ -closed  $\Rightarrow f(A)$  is  $v$ -open in  $Y$  since every  $\alpha$ -open set is  $v$ -open. Hence  $f$  is Almost contra  $v$ -closed.

Conversely Let  $A \subseteq X$  be closed  $\Rightarrow f(A)$  is  $v$ -open in  $Y$  since  $f: X \rightarrow Y$  is Almost contra  $v$ -closed  $\Rightarrow f(A)$  is  $\alpha$ -open in  $Y$  since every  $v$ -open set is  $\alpha$ -open. Hence  $f$  is Almost contra  $\alpha$ -closed.

**Theorem 3.8:** If  $f$  is closed and  $g$  is Almost contra  $v$ -closed then  $g \circ f$  is Almost contra  $v$ -closed.

**Proof:** Let  $A \subseteq X$  be closed  $\Rightarrow f(A)$  is closed in  $Y \Rightarrow g(f(A))$  is  $v$ -open in  $Z \Rightarrow g \circ f(A)$  is  $v$ -open in  $Z$ . Hence  $g \circ f$  is Almost contra  $v$ -closed.

**Theorem 3.9:** If  $f$  is closed and  $g$  is Almost contra  $r$ -closed then  $g \circ f$  is Almost contra  $v$ -closed.

**Proof:** Let  $A \subseteq X$  be closed  $\Rightarrow f(A)$  is closed in  $Y \Rightarrow g(f(A))$  is  $r$ -open in  $Z \Rightarrow g \circ f$  is  $v$ -open in  $Z$ . Hence  $g \circ f$  is Almost contra  $v$ -closed.

**Theorem 3.10:** If  $f$  is closed and  $g$  is Almost contra  $r\alpha$ -closed then  $g \circ f$  is Almost contra  $v$ -closed.

**Proof:** Let  $A \subseteq X$  be closed in  $X \Rightarrow f(A)$  is closed in  $Y \Rightarrow g(f(A))$  is  $r\alpha$ -open in  $Z \Rightarrow g(f(A))$  is  $v$ -open in  $Z \Rightarrow g \circ f$  is  $v$ -open in  $Z$ . Hence  $g \circ f$  is almost Almost contra  $v$ -closed.

**Theorem 3.11:** If  $f$  is  $r$ -closed and  $g$  is Almost contra  $v$ -closed then  $g \circ f$  is Almost contra  $v$ -closed.

**Proof:** Let  $A \subseteq X$  be closed  $\Rightarrow f(A)$  is  $r$ -closed in  $Y \Rightarrow g(f(A))$  is  $v$ -open in  $Z \Rightarrow g \circ f$  is  $v$ -open in  $Z$ . Hence  $g \circ f$  is Almost contra  $v$ -closed.

**Theorem 3.12:** If  $f$  is  $r$ -closed and  $g$  is Almost contra  $r$ -closed then  $g \circ f$  is Almost contra  $v$ -closed.

**Proof:** Let  $A \subseteq X$  be closed  $\Rightarrow f(A)$  is  $r$ -closed in  $Y \Rightarrow g(f(A))$  is  $r$ -open in  $Z \Rightarrow g \circ f$  is  $v$ -open in  $Z$ . Hence  $g \circ f$  is Almost contra  $v$ -closed.

**Theorem 3.13:** If  $f$  is  $r$ -closed and  $g$  is Almost contra  $r\alpha$ -closed then  $g \circ f$  is Almost contra  $v$ -closed.

**Proof:** Let  $A \subseteq X$  be closed in  $X \Rightarrow f(A)$  is  $r$ -closed in  $Y \Rightarrow g(f(A))$  is  $r\alpha$ -open in  $Z \Rightarrow g(f(A))$  is  $v$ -open in  $Z \Rightarrow g \circ f$  is  $v$ -open in  $Z$ . Hence  $g \circ f$  is Almost contra  $v$ -closed.

**Corollary 3.1:**

- If  $f$  is closed[ $r$ -closed] and  $g$  is Almost contra  $v$ -closed then  $g \circ f$  is Almost contra semi-closed and hence Almost contra  $\beta$ -closed.
- If  $f$  is closed[ $r$ -closed] and  $g$  is Almost contra  $r$ -closed then  $g \circ f$  is Almost contra semi-closed and hence Almost contra  $\beta$ -closed.
- If  $f$  is closed[ $r$ -closed] and  $g$  is Almost contra  $r\alpha$ -closed then  $g \circ f$  is Almost contra semi-closed and hence Almost contra  $\beta$ -closed.

**Theorem 3.14:** If  $f$  is Almost contra closed and  $g$  is  $v$ -open then  $g \circ f$  is Almost contra- $v$ -closed.

**Proof:** Let  $A \subseteq X$  be closed in  $X \Rightarrow f(A)$  is open in  $Y \Rightarrow g(f(A))$  is  $v$ -open in  $Z \Rightarrow g \circ f$  is  $v$ -open in  $Z$ . Hence  $g \circ f$  is Almost contra  $v$ -closed.

**Theorem 3.15:** If  $f$  is Almost contra closed and  $g$  is  $r$ -open then  $g \circ f$  is Almost contra- $v$ -closed.

**Proof:** Let  $A \subseteq X$  be closed in  $X \Rightarrow f(A)$  is open in  $Y \Rightarrow g(f(A))$  is  $r$ -open in  $Z \Rightarrow g(f(A))$  is  $v$ -open in  $Z \Rightarrow g \circ f$  is  $v$ -open in  $Z$ . Hence  $g \circ f$  is Almost contra  $v$ -closed.

**Theorem 3.16:** If  $f$  is Almost contra closed and  $g$  is  $r\alpha$ -open then  $g \circ f$  is Almost contra- $v$ -closed.

**Proof:** Let  $A \subseteq X$  be closed in  $X \Rightarrow f(A)$  is open in  $Y \Rightarrow g(f(A))$  is  $r\alpha$ -open in  $Z \Rightarrow g(f(A))$  is  $v$ -open in  $Z \Rightarrow g \circ f$  is  $v$ -open in  $Z$ . Hence  $g \circ f$  is Almost contra  $v$ -closed.

**Theorem 3.17:** If  $f$  is Almost contra- $r$ -closed and  $g$  is  $v$ -open then  $g \circ f$  is Almost contra- $v$ -closed.

**Proof:** Let  $A \subseteq X$  be closed in  $X \Rightarrow f(A)$  is  $r$ -open in  $Y \Rightarrow g(f(A))$  is  $v$ -open in  $Z \Rightarrow g \circ f$  is  $v$ -open in  $Z$ . Hence  $g \circ f$  is Almost contra  $v$ -closed.

**Theorem 3.18:** If  $f$  is Almost contra- $r$ -closed and  $g$  is  $r$ -open then  $g \circ f$  is Almost contra- $v$ -closed.

**Proof:** Let  $A \subseteq X$  be closed in  $X \Rightarrow f(A)$  is  $r$ -open in  $Y \Rightarrow g(f(A))$  is  $r$ -open in  $Z \Rightarrow g(f(A))$  is  $v$ -open in  $Z \Rightarrow g \circ f$  is  $v$ -open in  $Z$ . Hence  $g \circ f$  is Almost contra  $v$ -closed.

**Theorem 3.19:** If  $f$  is Almost contra- $r$ -closed and  $g$  is  $r\alpha$ -open then  $g \circ f$  is Almost contra- $v$ -closed.

**Proof:** Let  $A \subseteq X$  be closed in  $X \Rightarrow f(A)$  is  $r$ -open in  $Y \Rightarrow g(f(A))$  is  $r\alpha$ -open in  $Z \Rightarrow g(f(A))$  is  $v$ -open in  $Z \Rightarrow g \circ f$  is  $v$ -open in  $Z$ . Hence  $g \circ f$  is Almost contra  $v$ -closed.

**Corollary 3.2:**

- If  $f$  is Almost contra closed[Almost contra- $r$ -closed] and  $g$  is  $v$ -open then  $g \circ f$  is Almost contra-semi-closed and hence Almost contra  $\beta$ -closed.
- If  $f$  is Almost contra closed[Almost contra- $r$ -closed] and  $g$  is  $r$ -open then  $g \circ f$  is Almost contra-semi-closed and hence Almost contra  $\beta$ -closed.
- If  $f$  is Almost contra closed[Almost contra- $r$ -closed] and  $g$  is  $r\alpha$ -closed then  $g \circ f$  is Almost contra-semi-closed and hence Almost contra  $\beta$ -closed.

**Theorem 3.20:** If  $f: X \rightarrow Y$  is Almost contra  $v$ -closed, then  $f(A^\circ) \subseteq v(f(A))^\circ$

**Proof:** Let  $A \subseteq X$  be closed and  $f: X \rightarrow Y$  is Almost contra  $v$ -closed gives  $f(A^\circ)$  is  $v$ -open in  $Y$  and  $f(A^\circ) \subseteq f(A)$  which in turn gives  $v(f(A^\circ))^\circ \subseteq v(f(A))^\circ$  --- (1)  
Since  $f(A^\circ)$  is  $v$ -open in  $Y$ ,  $v(f(A^\circ))^\circ = f(A^\circ)$  ----- (2)  
combining (1) and (2) we have  $f(A^\circ) \subseteq v(f(A))^\circ$  for every subset  $A$  of  $X$ .

**Remark 2:** Converse is not true in general.

**Corollary 3.3:** If  $f: X \rightarrow Y$  is Almost contra  $r$ -closed, then  $f(A^\circ) \subset v(f(A))^\circ$

**Proof:** Let  $A \subset X$  be closed and  $f: X \rightarrow Y$  is Almost contra  $r$ -closed gives  $f(A)^\circ$  is  $r$ -open in  $Y$  and  $f(A^\circ) \subset f(A)$  which in turn gives  $v(f(A^\circ))^\circ \subset v(f(A))^\circ$  -----(1)  
Since  $f(A^\circ)$  is  $v$ -open in  $Y$ ,  $v(f(A^\circ))^\circ = f(A^\circ)$  -----(2)  
combining (1) and (2) we have  $f(A^\circ) \subset v(f(A))^\circ$  for every subset  $A$  of  $X$ .

**Theorem 3.21:** If  $f: X \rightarrow Y$  is Almost contra  $v$ -closed and  $A \subset X$  is closed,  $f(A)$  is  $\tau_v$ -open in  $Y$ .

**Proof:** Let  $A \subset X$  be closed and  $f: X \rightarrow Y$  is Almost contra  $v$ -closed  $\Rightarrow f(A^\circ) \subset v(f(A))^\circ \Rightarrow f(A) \subset v(f(A))^\circ$ , since  $f(A) = f(A^\circ)$ . But  $v(f(A))^\circ \subset f(A)$ . Combining we get  $f(A) = v(f(A))^\circ$ . Therefore  $f(A)$  is  $\tau_v$ -open in  $Y$ .

**Corollary 3.4:** If  $f: X \rightarrow Y$  is Almost contra  $r$ -closed, then  $f(A)$  is  $\tau_v$ -open in  $Y$  if  $A$  is  $r$ -closed set in  $X$ .

**Proof:** Let  $A \subset X$  be  $r$ -closed and  $f: X \rightarrow Y$  is Almost contra  $r$ -closed  $\Rightarrow f(A^\circ) \subset r(f(A))^\circ \Rightarrow f(A^\circ) \subset v(f(A))^\circ$  (by theorem 3.20)  $\Rightarrow f(A) \subset v(f(A))^\circ$ , since  $f(A) = f(A^\circ)$ . But  $v(f(A))^\circ \subset f(A)$ . Combining we get  $f(A) = v(f(A))^\circ$ . Hence  $f(A)$  is  $\tau_v$ -open in  $Y$ .

**Theorem 3.22:** If  $v(A)^\circ = r(A)^\circ$  for every  $A \subset Y$ , then the following are equivalent:

- $f: X \rightarrow Y$  is Almost contra  $v$ -closed map
- $f(A^\circ) \subset v(f(A))^\circ$

**Proof:** (a)  $\Rightarrow$  (b) follows from theorem 3.20.

(b)  $\Rightarrow$  (a) Let  $A$  be any  $r$ -closed set in  $X$ , then  $f(A) = f(A^\circ) \subset v(f(A))^\circ$  by hypothesis. We have  $f(A) \subset v(f(A))^\circ$ . Combining we get  $f(A) = v(f(A))^\circ = r(f(A))^\circ$  [by given condition] which implies  $f(A)$  is  $r$ -open and hence  $v$ -open. Thus  $f$  is Almost contra  $v$ -closed.

**Theorem 3.23:**  $f: X \rightarrow Y$  is Almost contra  $v$ -closed iff for each subset  $S$  of  $Y$  and each open set  $U$  containing  $f^{-1}(S)$ , there is an  $v$ -closed set  $V$  of  $Y$  such that  $S \subset V$  and  $f^{-1}(V) \subset U$ .

**Remark 3:** Composition of two Almost contra  $v$ -closed maps is not Almost contra  $v$ -closed in general.

**Theorem 3.24:** Let  $X, Y, Z$  be topological spaces and every  $v$ -open set is closed [ $r$ -closed] in  $Y$ . Then the composition of two Almost contra  $v$ -closed [Almost contra  $r$ -closed] maps is Almost contra  $v$ -closed.

**Proof:** (a) Let  $f: X \rightarrow Y$  and  $g: Y \rightarrow Z$  be Almost contra  $v$ -closed maps. Let  $A$  be any closed set in  $X \Rightarrow f(A)$  is  $v$ -open in  $Y \Rightarrow f(A)$  is closed in  $Y$  (by assumption)  $\Rightarrow g(f(A))$  is  $v$ -open in  $Z \Rightarrow g \circ f(A)$  is  $v$ -open in  $Z$ . Therefore  $g \circ f$  is Almost contra  $v$ -closed.

(b) Let  $f: X \rightarrow Y$  and  $g: Y \rightarrow Z$  be Almost contra  $v$ -closed maps. Let  $A$  be any closed set in  $X \Rightarrow f(A)$  is  $r$ -open in  $Y \Rightarrow f(A)$  is  $v$ -open in  $Y \Rightarrow f(A)$  is  $r$ -closed in  $Y$  (by assumption)  $\Rightarrow f(A)$  is closed in  $Y$  (by assumption)  $\Rightarrow g(f(A))$  is  $r$ -open in  $Z \Rightarrow g \circ f(A)$  is  $v$ -open in  $Z$ . Therefore  $g \circ f$  is Almost contra  $v$ -closed.

**Theorem 3.25:** Let  $X, Y, Z$  be topological spaces and  $Y$  is discrete topological space in  $Y$ . Then the composition of two Almost contra  $v$ -closed [Almost contra  $r$ -closed] maps is Almost contra  $v$ -closed.

**Theorem 3.26:** If  $f: X \rightarrow Y$  is  $g$ -closed,  $g: Y \rightarrow Z$  is Almost contra  $v$ -closed [Almost contra  $r$ -closed] and  $Y$  is  $T_{1/2}$  [ $r$ - $T_{1/2}$ ] then  $g \circ f$  is Almost contra  $v$ -closed.

**Proof:** (a) Let  $A$  be a closed set in  $X$ . Then  $f(A)$  is  $g$ -closed set in  $Y \Rightarrow f(A)$  is closed in  $Y$  as  $Y$  is  $T_{1/2} \Rightarrow g(f(A))$  is  $v$ -open in  $Z$  since  $g$  is Almost contra  $v$ -closed  $\Rightarrow g \circ f(A)$  is  $v$ -open in  $Z$ . Hence  $g \circ f$  is Almost contra  $v$ -closed.

(b) Let  $A$  be a closed set in  $X$ . Then  $f(A)$  is  $g$ -closed set in  $Y \Rightarrow f(A)$  is closed in  $Y$  as  $Y$  is  $T_{1/2} \Rightarrow g(f(A))$  is  $r$ -open in  $Z$  since  $g$  is Almost contra  $r$ -closed  $\Rightarrow g \circ f(A)$  is  $v$ -open in  $Z$ . Hence  $g \circ f$  is Almost contra  $v$ -closed.

**Corollary 3.5:** If  $f: X \rightarrow Y$  is  $g$ -open,  $g: Y \rightarrow Z$  is Almost contra  $v$ -closed [Almost contra  $r$ -closed] and  $Y$  is  $T_{1/2}$  [ $r$ - $T_{1/2}$ ] then  $g \circ f$  is Almost contra  $p$ -closed and hence Almost contra  $\beta$ -closed.

**Theorem 3.27:** If  $f: X \rightarrow Y$  is  $rg$ -open,  $g: Y \rightarrow Z$  is Almost contra  $v$ -closed [Almost contra  $r$ -closed] and  $Y$  is  $r$ - $T_{1/2}$ , then  $g \circ f$  is Almost contra  $v$ -closed.

**Proof:** Let  $A$  be a closed set in  $X$ . Then  $f(A)$  is  $rg$ -closed in  $Y \Rightarrow f(A)$  is  $r$ -closed in  $Y$  since  $Y$  is  $r$ - $T_{1/2} \Rightarrow f(A)$  is closed in  $Y$  since every  $r$ -closed set is closed  $\Rightarrow g(f(A))$  is  $v$ -open in  $Z \Rightarrow g \circ f(A)$  is  $v$ -open in  $Z$ . Hence  $g \circ f$  is Almost contra  $v$ -closed.

**Corollary 3.6:** If  $f: X \rightarrow Y$  is  $rg$ -open,  $g: Y \rightarrow Z$  is Almost contra  $v$ -closed [Almost contra  $r$ -closed] and  $Y$  is  $r$ - $T_{1/2}$ , then  $g \circ f$  is Almost contra semi-closed and hence Almost contra  $\beta$ -closed.

**Theorem 3.28:** If  $f: X \rightarrow Y, g: Y \rightarrow Z$  be two mappings such that  $g \circ f$  is Almost contra  $v$ -closed [Almost contra  $r$ -closed] then the following statements are true.

- If  $f$  is continuous [ $r$ -continuous] and surjective then  $g$  is Almost contra  $v$ -closed.
- If  $f$  is  $g$ -continuous, surjective and  $X$  is  $T_{1/2}$  then  $g$  is Almost contra  $v$ -closed.
- If  $f$  is  $rg$ -continuous, surjective and  $X$  is  $r$ - $T_{1/2}$  then  $g$  is Almost contra  $v$ -closed.

**Proof:** (a) Let  $A$  be a closed set in  $Y \Rightarrow f^{-1}(A)$  is closed in  $X \Rightarrow (g \circ f)(f^{-1}(A))$  is  $v$ -open in  $Z \Rightarrow g(A)$  is  $v$ -open in  $Z$ . Hence  $g$  is Almost contra  $v$ -closed.

(b) Let  $A$  be a closed set in  $Y \Rightarrow f^{-1}(A)$  is  $g$ -closed in  $X \Rightarrow f^{-1}(A)$  is closed in  $X$  [since  $X$  is  $T_{1/2}$ ]  $\Rightarrow (g \circ f)(f^{-1}(A))$  is  $v$ -open in  $Z \Rightarrow g(A)$  is  $v$ -open in  $Z$ . Hence  $g$  is Almost contra  $v$ -closed.

(c) Let  $A$  be a closed set in  $Y \Rightarrow f^{-1}(A)$  is  $g$ -closed in  $X \Rightarrow f^{-1}(A)$  is closed in  $X$  [since  $X$  is  $r$ - $T_{1/2}$ ]  $\Rightarrow (g \circ f)(f^{-1}(A))$  is  $v$ -open in  $Z \Rightarrow g(A)$  is  $v$ -open in  $Z$ . Hence  $g$  is Almost contra  $v$ -closed.

**Corollary 3.7:** If  $f: X \rightarrow Y$ ,  $g: Y \rightarrow Z$  be two mappings such that  $g \circ f$  is Almost contra  $v$ -closed [Almost contra  $r$ -closed] then the following statements are true.

- If  $f$  is continuous [ $r$ -continuous] and surjective then  $g$  is Almost contra semi-closed and hence Almost contra  $\beta$ -closed.
- If  $f$  is  $g$  continuous, surjective and  $X$  is  $T_{1/2}$  then  $g$  is Almost contra semi-closed and hence Almost contra  $\beta$ -closed.
- If  $f$  is  $rg$ -continuous, surjective and  $X$  is  $rT_{1/2}$  then  $g$  is Almost contra semi-closed and hence Almost contra  $\beta$ -closed.

**Theorem 3.29:** If  $f: X \rightarrow Y$  is Almost contra  $v$ -closed and  $A$  is a closed set of  $X$  then  $f_A: (X, \tau(A)) \rightarrow (Y, \sigma)$  is Almost contra  $v$ -closed.

**Proof:** (a) Let  $F$  be a closed set in  $A$ . Then  $F = A \cap E$  for some closed set  $E$  of  $X$  and so  $F$  is closed in  $X \Rightarrow f(A)$  is  $v$ -open in  $Y$ . But  $f(F) = f_A(F)$ . Therefore  $f_A$  is Almost contra  $v$ -closed.

**Theorem 3.30:** If  $f: X \rightarrow Y$  is Almost contra  $r$ -closed and  $A$  is a closed set of  $X$  then  $f_A: (X, \tau(A)) \rightarrow (Y, \sigma)$  is Almost contra  $v$ -closed.

**Proof:** Let  $F$  be a closed set in  $A$ . Then  $F = A \cap E$  for some closed set  $E$  of  $X$  and so  $F$  is closed in  $X \Rightarrow f(A)$  is  $r$ -open in  $Y \Rightarrow f(A)$  is  $v$ -open in  $Y$ . But  $f(F) = f_A(F)$ . Therefore  $f_A$  is Almost contra  $v$ -closed.

**Corollary 3.8:** If  $f: X \rightarrow Y$  is Almost contra  $v$ -closed [Almost contra  $r$ -closed] and  $A$  is a closed set of  $X$  then  $f_A: (X, \tau(A)) \rightarrow (Y, \sigma)$  is Almost contra semi-closed and hence Almost contra  $\beta$ -closed.

**Theorem 3.31:** If  $f: X \rightarrow Y$  is Almost contra  $v$ -closed,  $X$  is  $T_{1/2}$  and  $A$  is  $g$ -closed set of  $X$  then  $f_A: (X, \tau(A)) \rightarrow (Y, \sigma)$  is Almost contra  $v$ -closed.

**Proof:** Let  $F$  be a closed set in  $A$ . Then  $F = A \cap E$  for some closed set  $E$  of  $X$  and so  $F$  is closed in  $X \Rightarrow f(A)$  is  $v$ -open in  $Y$ . But  $f(F) = f_A(F)$ . Therefore  $f_A$  is Almost contra  $v$ -closed.

**Theorem 3.32:** If  $f: X \rightarrow Y$  is Almost contra- $r$ -closed,  $X$  is  $T_{1/2}$  and  $A$  is  $g$ -closed set of  $X$  then  $f_A: (X, \tau(A)) \rightarrow (Y, \sigma)$  is Almost contra  $v$ -closed.

**Proof:** Let  $F$  be a closed set in  $A$ . Then  $F = A \cap E$  for some closed set  $E$  of  $X$  and so  $F$  is closed in  $X \Rightarrow f(A)$  is  $r$ -open in  $Y \Rightarrow f(A)$  is  $v$ -open in  $Y$ . But  $f(F) = f_A(F)$ . Therefore  $f_A$  is Almost contra  $v$ -closed.

**Corollary 3.9:** If  $f: X \rightarrow Y$  is Almost contra  $v$ -closed [Almost contra  $r$ -closed],  $X$  is  $T_{1/2}$ ,  $A$  is  $g$ -closed set of  $X$  then  $f_A: (X, \tau(A)) \rightarrow (Y, \sigma)$  is Almost contra semi-closed and hence Almost contra  $\beta$ -closed.

**Theorem 3.33:** If  $f_i: X_i \rightarrow Y_i$  be Almost contra  $v$ -closed [Almost contra  $r$ -closed] for  $i = 1, 2$ . Let  $f: X_1 \times X_2 \rightarrow Y_1 \times Y_2$  be defined as  $f(x_1, x_2) = (f_1(x_1), f_2(x_2))$ . Then  $f: X_1 \times X_2 \rightarrow Y_1 \times Y_2$  is Almost contra  $v$ -closed.

**Proof:** Let  $U_1 \times U_2 \subseteq X_1 \times X_2$  where  $U_i$  is closed in  $X_i$  for  $i = 1, 2$ . Then  $f(U_1 \times U_2) = f_1(U_1) \times f_2(U_2)$  is  $v$ -open set in  $Y_1 \times Y_2$ . Then  $f(U_1 \times U_2)$  is  $v$ -open set in  $Y_1 \times Y_2$ . Hence  $f$  is Almost contra  $v$ -closed.

**Corollary 3.10:** If  $f_i: X_i \rightarrow Y_i$  be Almost contra  $v$ -closed [Almost contra  $r$ -closed] for  $i = 1, 2$ . Let  $f: X_1 \times X_2 \rightarrow Y_1 \times Y_2$  be defined as  $f(x_1, x_2) = (f_1(x_1), f_2(x_2))$ , then  $f: X_1 \times X_2 \rightarrow Y_1 \times Y_2$  is Almost contra semi-closed and hence Almost contra  $\beta$ -closed.

**Theorem 3.34:** Let  $h: X \rightarrow X_1 \times X_2$  be Almost contra  $v$ -closed. Let  $f_i: X \rightarrow X_i$  be defined as  $h(x) = (x_1, x_2)$  and  $f_i(x) = x_i$ . Then  $f_i: X \rightarrow X_i$  is Almost contra  $v$ -closed for  $i = 1, 2$ .

**Proof:** Let  $U_1$  be closed in  $X_1$ , then  $U_1 \times X_2$  is closed in  $X_1 \times X_2$ , and  $h(U_1 \times X_2)$  is  $v$ -open in  $X$ . But  $f_1(U_1) = h(U_1 \times X_2)$ , therefore  $f_1$  is Almost contra  $v$ -closed. Similarly we can show that  $f_2$  is also Almost contra  $v$ -closed and thus  $f_i: X \rightarrow X_i$  is Almost contra  $v$ -closed for  $i = 1, 2$ .

**Corollary 3.11:** Let  $h: X \rightarrow X_1 \times X_2$  be Almost contra  $v$ -closed. Let  $f_i: X \rightarrow X_i$  be defined as  $h(x) = (x_1, x_2)$  and  $f_i(x) = x_i$ . Then  $f_i: X \rightarrow X_i$  is Almost contra semi-closed and hence Almost contra  $\beta$ -closed for  $i = 1, 2$ .

## 4. CONCLUSION

In this paper we introduced the concept of almost contra  $v$ -closed mappings, studied their basic properties and the interrelationship between other closed maps.

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